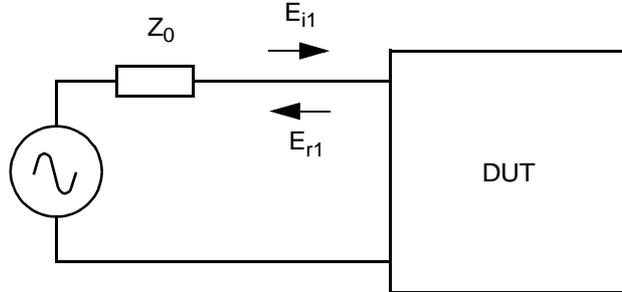


# RF-Microwaves formulas - 1-port systems

## s-parameters:

Considering a voltage source feeding into the DUT with a source impedance of  $Z_0$ .



The voltage into the DUT is composed of 2 parts, an incident wave  $E_{i1}$  and a reflected one  $E_{r1}$  characterized by voltage and current :

$$V_1 = E_{i1} + E_{r1} \quad I_1 = \frac{E_{i1} - E_{r1}}{Z_0} \quad (1)$$

Noting that the DUT is connected to the source by a length of transmission line of characteristic impedance  $Z_0$  (possibly vanishingly small as in the example above), the voltage along the line is the sum of the 2 waves ( $V_1$ ) while the current, because the waves are travelling in opposite direction is the difference of the 2 waves, divided by  $Z_0$ . This is further described in the Appendix at the end of this document.

By defining the following normalized notation:

$$a_1 = \frac{E_{i1}}{\sqrt{Z_0}} \quad b_1 = \frac{E_{r1}}{\sqrt{Z_0}} \quad (2)$$

$a_1$  and  $b_1$  become the square root of the forward and reflected powers. Note thus that the square of  $a_1$  and  $b_1$  have the dimension of power. We can write a function  $s_{11}$  such that:

$$b_1 = s_{11}a_1 \quad (3)$$

relating these 2 voltage waves and  $s_{11}$  is:

$$s_{11} = \frac{b_1}{a_1} \quad (4)$$

## Input impedance:

The input impedance can be expressed either as a rectangular value ( $Z_L = R + jX$  with  $R$  being the real part of  $Z_L$  and  $X$  the imaginary part of  $Z_L$ ) or as a polar value ( $Z_L = M \angle P$  here  $M$  is the magnitude of  $Z_L$  and  $P$  its phase angle) :

$$Z_L = \frac{1 + s_{11}}{1 - s_{11}} \cdot Z_0 \quad (5)$$

### Reflection coefficient:

$$\Gamma = \rho \angle \theta = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Y_0 - Y_L}{Y_0 + Y_L} \quad (6)$$

The **reflection coefficient** *gamma* represents the quality of the impedance match between the source and the measured load. It is a complex quantity, with magnitude *rho* and angle *theta*.

The **reflection coefficient** is small for good matches. The **reflection coefficient** takes values from  $-1$  for shorts, stays negative for loads  $< Z_0$ , is zero for perfect matches, is positive for loads  $> Z_0$ , and reaches  $+1$  for open loads.

Using normalized impedances:

$$\Gamma = \rho \angle \theta = \frac{Z - 1}{Z + 1} = \frac{1 - Y}{1 + Y} \quad \text{and} \quad Z = \frac{1 + \Gamma}{1 - \Gamma} \quad (7)$$

From equ.(5), it follows that  $s_{11}$  is:

$$s_{11} = \frac{Z - 1}{Z + 1} = \Gamma \quad (8)$$

Here we see that  $s_{11}$  and  $\Gamma$  (the **reflection coefficient**) are one and the same.

### Return Loss:

Considering the **reflection coefficient** as:

$$\Gamma = \rho \angle \theta \quad (9)$$

the **Return Loss** (expressed in dB) makes use of  $\rho$  to be:

$$L_r = 20 \log\left(\frac{1}{\rho}\right) = -20 \log(\rho) \quad (10)$$

The higher the absolute value of the **Return Loss**, the better the match. The **Return Loss** is a measure of the power that is not absorbed by the load and is therefore returned to the source. So large absolute values of the **Return Loss** (i.e.:  $> |20|$  dB) imply a good match.

### VSWR:

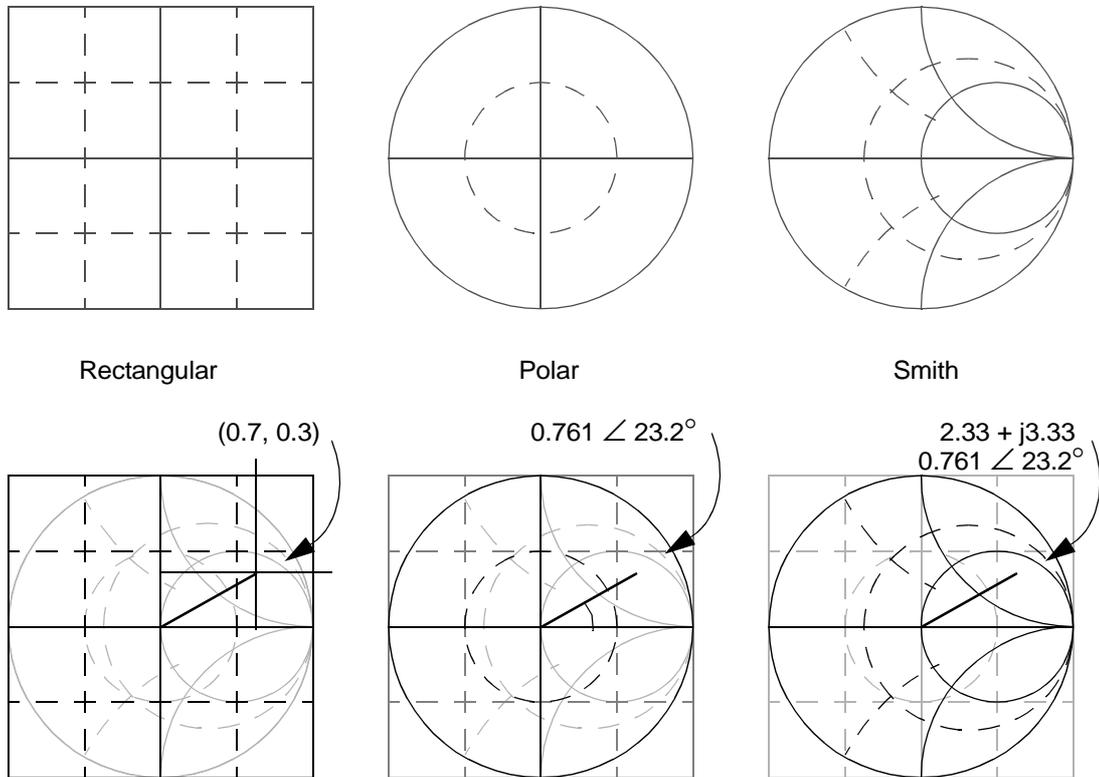
Just as for the Return Loss, the **VSWR** (dimensionless) is calculated from  $\rho$ :

$$VSWR = \frac{1 + \rho}{1 - \rho} = \frac{1 + |s_{11}|}{1 - |s_{11}|} \quad (11)$$

The **VSWR** is useful to estimate (calculate) the peak voltage that can be found on a line under non-ideal match conditions. It is also an common measure of the “goodness” of a match. A perfect match is characterized by a VSWR of 1, while a short or open circuit produces a VSWR of  $\infty$ .

## Charts and Smith Chart:

There are three types of charts commonly used to represent complex values in RF work. The Rectangular Chart is used for rectangular values ( $R + jX$ ), the Polar Chart for polar values ( $M\angle P$ ) and the Smith Chart for both of these, and S-parameters also (and a many other values too).



One interesting observation is that with normalized parameters, all 3 charts can be overlaid, as shown on the 3 figures above, and the same vector can be read in several manners off their respective axes.

On a **Smith Chart**, one plots **normalized impedances** if referring to the curved axes, it is also possible with a protractor and a ruler to enter s-parameters as magnitude and angle, as would be done on the polar plot. The complete smith chart has an angle scale all around it for that purpose, and at the bottom of the chart, there usually is a graduated scale of the same diameter as the chart, on which it is possible to report and read the polar magnitude of the vector.

### Example:

For the series circuit of  $L = 26.5 \text{ nH}$  and  $R = 116.5 \Omega$  at  $1 \text{ GHz}$ .

We get :  $Z = 116.5 + j166.5$ , normalized to  $50 \Omega$ :  $Z_n = 2.33 + j3.33$

Calculating  $S_{11}$  which is equal to  $\Gamma$ :

$$S_{11} = \Gamma = \frac{Z_n - 1}{Z_n + 1} = 0.7 + j0.3 = 0.761 \angle 23.2^\circ \quad (12)$$

Other quantities of interest:

$$VSWR = \frac{1 + \rho}{1 - \rho} = \frac{1 + 0.761}{1 - 0.761} = 7.38 \quad (13)$$

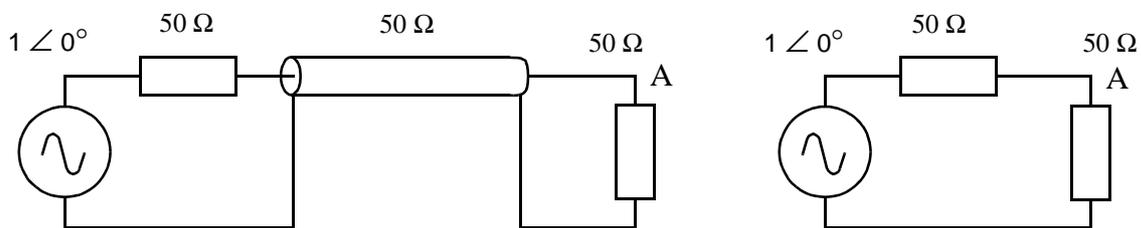
$$ReturnLoss = 20\log\left(\frac{1}{\rho}\right) = 2.37 \text{ dB} \quad (14)$$

These values have been identified on the three graphs above.

### Appendix:

Let's consider 3 obvious cases of a segment of transmission line, either properly terminated, or open or shorted, and let's apply equ.(1) at point A, at the end of the transmission line, noting that the transmission line can be any length, included none.

#### Line properly terminated:



Note that either schematic can be used for the following observations.

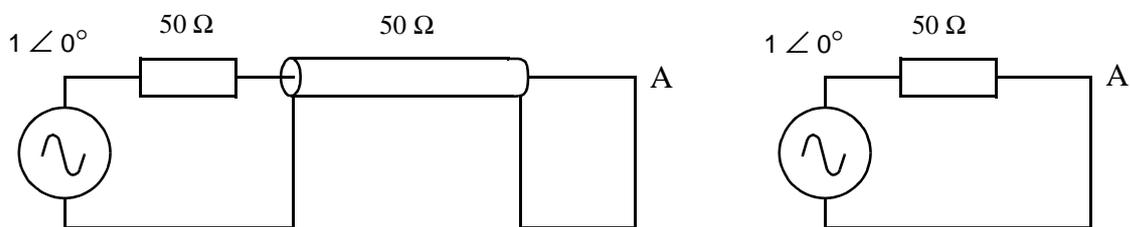
Divider action sets the voltage at A at:  $0.5\angle 0$  V

Ohm's law says that  $I_A$  is:  $\frac{1\angle 0}{100} = 10\angle 0$  mA

We know that in a properly matched system there will be a forward wave but no reflected wave. So according to equ.(1) at point A:

$$V = 0.5\angle 0 + 0 = 0.5\angle 0 \text{ V} \quad I = \frac{0.5\angle 0 - 0}{50} = 10\angle 0 \text{ mA} \quad (15)$$

#### Shorted line:

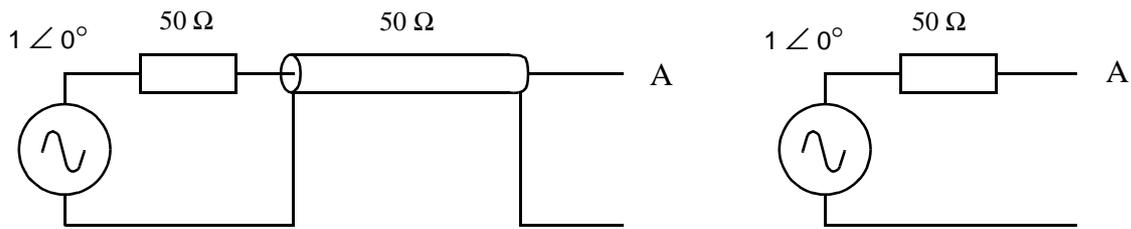


By inspection the voltage at point A is zero and the current is:  $\frac{1\angle 0}{50} = 20\angle 0$  mA.

We know that in a shorted line, there will be a forward wave and a reflected one but with reverse polarity. So according to equ.(1) at point A:

$$V = 0.5\angle 0 - 0.5\angle 0 = 0 \text{ V} \quad I = \frac{0.5\angle 0 - (-0.5\angle 0)}{50} = 20 \text{ mA} \quad (16)$$

### Opened line:



By inspection the voltage at point A is  $1 \angle 0$  V and the current 0.

We know that in an opened line, there will be a forward wave and a reflected one of the same polarity. So according to equ.(1) at point A:

$$V = 0.5 \angle 0 + 0.5 \angle 0 = 1 \angle 0 \text{ V} \quad I = \frac{0.5 \angle 0 - 0.5 \angle 0}{50} = 0 \quad (17)$$

Note: One last question might be: why did we choose a value of 0.5 V for the forward and reflected waves? Well simply because it is the only value that fits all cases, but there is a better and simpler answer; in the matched case, where there is no reflection, this is the value predicted for the forward wave by Ohm's law and by the maximum power transfer theorem.

Note : Une version en Français de cet article est disponible sur :  
[http://www.pilloud/net/op\\_web/tek.html#05](http://www.pilloud/net/op_web/tek.html#05)

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